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No. 461

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CONTRIBUTION TO THE SYSTEMATIC INVESTIGATION  
ON JOUKOWSKY PROFILES

By Gottfried Loew

From "Zeitschrift für Flugtechnik und Motorluftschiffahrt"  
November 28, 1927

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Washington  
April, 1928

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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

TECHNICAL MEMORANDUM NO. 461.

CONTRIBUTION TO THE SYSTEMATIC INVESTIGATION  
OF JOUKOWSKY PROFILES.\*

By Gottfried Loew.

This article resulted from the need of showing, in a simple way, how the aerodynamic properties of airfoils are affected by the shape of their profiles. No general solution of this problem could be found, since the profile shapes cannot ordinarily be expressed by simple mathematical formulas. This advantage is possessed only by the Joukowsky profiles and this discussion of the problem is therefore limited to them. Oskar Schrenk published in this magazine ("Zeitschrift für Flugtechnik und Motorluftschiffahrt," May 28, 1927, pp. 225-230) an article entitled "Systematische Untersuchungen an Joukowsky-Profilen" (For translation, see N.A.C.A. Technical Memorandum No. 422, "Systematic Investigation of Joukowsky Wing Sections"). (See also "Ergebnisse der Aerodynamischen Versuchsanstalt zu Göttingen," Report III.) The present investigation differs from the abovementioned one only in the form of presentation. The 74th and 79th reports of the D.V.L. ("Deutsche Versuchsanstalt für Luftfahrt") also relate to this work (N.A.C.A. Technical Memorandums Nos. 456 and 457).

To what extent the constants assumed by Martin Schrenk really

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\*"Ein Beitrag zur Systematik der Joukowsky-Profile" in "Zeitschrift für Flugtechnik und Motorluftschiffahrt," November 28, 1927, pp. 517-522.

THE  
UNITED STATES OF AMERICA  
DEPARTMENT OF THE INTERIOR  
BUREAU OF LAND MANAGEMENT  
WASHINGTON, D. C. 20250

TO: [Name] [Address] [City] [State] [Zip]  
FROM: [Name] [Address] [City] [State] [Zip]  
SUBJECT: [Subject]  
RE: [Reference]  
[Text of the letter follows, containing several paragraphs of text, some of which are partially obscured by the scanning process. The text appears to be a formal communication, possibly a letter of transmittal or a report, discussing land management issues. It includes references to various documents and sections, such as "Section 1", "Section 2", "Section 3", "Section 4", "Section 5", "Section 6", "Section 7", "Section 8", "Section 9", "Section 10", "Section 11", "Section 12", "Section 13", "Section 14", "Section 15", "Section 16", "Section 17", "Section 18", "Section 19", "Section 20", "Section 21", "Section 22", "Section 23", "Section 24", "Section 25", "Section 26", "Section 27", "Section 28", "Section 29", "Section 30", "Section 31", "Section 32", "Section 33", "Section 34", "Section 35", "Section 36", "Section 37", "Section 38", "Section 39", "Section 40", "Section 41", "Section 42", "Section 43", "Section 44", "Section 45", "Section 46", "Section 47", "Section 48", "Section 49", "Section 50", "Section 51", "Section 52", "Section 53", "Section 54", "Section 55", "Section 56", "Section 57", "Section 58", "Section 59", "Section 60", "Section 61", "Section 62", "Section 63", "Section 64", "Section 65", "Section 66", "Section 67", "Section 68", "Section 69", "Section 70", "Section 71", "Section 72", "Section 73", "Section 74", "Section 75", "Section 76", "Section 77", "Section 78", "Section 79", "Section 80", "Section 81", "Section 82", "Section 83", "Section 84", "Section 85", "Section 86", "Section 87", "Section 88", "Section 89", "Section 90", "Section 91", "Section 92", "Section 93", "Section 94", "Section 95", "Section 96", "Section 97", "Section 98", "Section 99", "Section 100".]

hold good, can be readily learned from the diagrams.

The shape of a Joukowski profile is determined by the thickness parameter  $d/l$  and the camber parameter  $f/l$ . In the representation, one of the parameters was always assumed to be constant, while the other varied (Fig. 1). Diagrams were thus obtained for a thickness group  $d/l = \text{constant}$  with variable camber or vice versa. The aerodynamic properties were represented by a function  $c_a = f(c_w)$ ,  $c_a = f(a)$ ,  $c_a = f(c_w/c_a)$ , or some similar one, according to the particular point of view. Hence there were three variables to be represented. This was done by plotting the values of  $f/l$ , in any desired scale, on the abscissas of a rectangular system of coordinates. The values of  $c_a$  were plotted on the axis of the ordinates. Obviously there is a point  $P_1(c_{a1}, f/l_1)$  for each value of  $c_{wI}$ . In normal regions of like camber, i.e. with one and the same profile, the same  $c_{wI}$  value will occur with a different  $c_a$  value. Still another point  $P_1'(c_{a1}, f/l_1)$  corresponds therefore to our  $c_{wI}$  value. In the case of a neighboring camber, the same  $c_{wI}$ , if it has a normal value, will appear in a similar manner at the points  $P_2(c_{a2}, f/l_2)$  and  $P_2'(c_{a2}', f/l_2)$ . In this way, points can be found for the  $c_{wI}$  values in the field on the different parallels to the ordinate axis, each parallel indicating a camber and therefore a profile. All these points can then be connected by a curve, which shows how, with increasing camber, a constant  $c_w$  value occasionally coordinates with another  $c_a$  value. If

[illegible]

such curves,  $c_w = \text{constant}$ , be drawn for the most divergent values of  $c_w$ , there is produced what may be called a topographic map or diagram.

With this method of representation, we may speak of the curves  $c_w = \text{constant}$  as lines of elevation or stratification. The curve of minimum drag surrounds the peak. The distance between the consecutive lines (the slope of the field) indicates the greater or less rapid increase in the drag, while the steep declivities indicate a separation of the flow. Sections through the field parallel to the axis of the ordinates give the polars for the camber or thickness represented on the abscissas. In this way intermediate profiles can be determined. The comparison of different charts representing individual thickness groups is more valuable and instructive.

The fundamental tendency of these fields is identical. They have a certain longitudinal axis, along which the intervals between the curves of constant  $c_w$  reach their maximum. From a spatial viewpoint, the most gradual slope of the drag field lies on this axis. In this direction the increase in the profile drag, with increasing  $c_a$ , is the smallest. The inclination of this axis is about the same for all fields. The boundary curves for the  $c_{a \text{ max}}$  of each profile likewise approximately coincide for thin, thick and very thick profiles ( $d/l = 0.10, 0.15, 0.20$ ). The thin profile remains about  $c_a = 0.2$  behind the others. The lift increase of a profile is therefore predominantly a function

of its camber and its influence is in fact greater in thin profiles than in thick ones. The lower region is practically covered for nearly all profiles. The minimum profile drag for the thin profile lies at  $c_a = 0$  and  $f/l = 0$  and shifts with increasing thickness to about  $c_a = 0.45$  and  $f/l = 0.05$ . For  $d/l = 0.05$ , the absolute coefficient of drag is  $c_{wp} = 0.065$ . For  $d/l = 0.2$ ,  $c_{wp} = 0.12$ , which is nearly 100% worse.

For lack of space, only a few pages of the whole report can be published here, so that only a few of the examples can be given. Likewise some of the diagrams must be omitted, which were drawn for one and the same profile group, both for the simple profile drag and for the total drag of the Göttingen airfoil sections.

Figure 2 shows the profile-drag coefficients for a thin profile covering all the cambers investigated. The fundamental tendency of the field is a very stable and steady one, probably due somewhat to the influence of the few available data. Since the field has a sharp ridge and steep slopes, the profiles show a very good optimum, but a very small good region. The separation takes place very uniformly both in the upper and in the lower region.

Quite a different picture is presented by Figure 3, which applies to profile thicknesses much used on engine-driven airplanes. They have a convenient constructional thickness, without

being too thick. The field has a very unstable character. The  $c_{a \max}$  is very high and climbs very rapidly with small cambers, but slower with large cambers. The separation processes are very abrupt in the lower region for ordinary profiles, but are more gradual for larger cambers. The minimum forms a distinct peak around the origin or zero point, but a flatter minimum is developed again over profile 580. It is obvious that the data for profile 541 were affected by some disturbance, since the whole field is distorted. I have heard that this was due to imperfections in the model. Furthermore, a symmetry of the polars of the symmetrical profile ( $f/l = 0$ ) is to be expected.

Figure 4 represents a rectification of field 10. Despite the surprising uniformity, only a few measurements were made. The values of the profile 429 were arranged symmetrically with respect to the origin. Measurements with profile 541 were omitted, since they were unreliable. Lastly, care was taken that the intervals between the curves diminished uniformly, which is a necessary condition for a steady course of the polars. A negative camber denotes an inverted airfoil of positive camber, which is measured in the normal direction. The field was plotted only for values up to  $c_{wp} = 0.02$ . The corrections from subsequent check tests, the results of which are given in the third report of the Göttingen A.V.A. ("Ergebnisse der Aerodynamischen Versuchsanstalt zu Göttingen, 3 Lieferung"), greatly improve the field of departure 13.



Figure 5 was plotted for different thicknesses with the same camber. The field is to be regarded as a vertical section through the superposed camber fields. It shows a somewhat improved character. The  $c_{a \max}$  is hardly affected by the thickness. The lower region, however, is favorably affected by the thickness. The minimum  $c_{wp}$  coefficient is obtained with thin airfoils.

Figures 6 and 7 afford a comparison of the moment coefficients in theory and in practice. In the third Göttingen report, already referred to, the theoretical formulas are so transformed, with the aid of a numerical table, that the theoretical calculations can be made for a perfectly definite Joukowski profile.

On the basis of such a table, the Göttingen investigators plotted theoretical lines of moment in their polar diagrams. The fields of practical and theoretical values were both plotted for the thickness  $d/l = 0.1$ . The camber appears again on the abscissa and the lift coefficients on the ordinate, while lines of constant-moment coefficients appear in the field. The Göttingen theoretical lines of moment were taken as the basis for plotting the theoretical moment field. The curves themselves were calculated, however, for the profiles 545 and 431. It was found, as also at Göttingen, that straight lines occur for small cambers, while curved lines are developed for large cambers. The location of the points is affected by the slightest inaccuracies in calculation. The tendency of the field in the region undisturbed

by separation phenomena is the same. In the region of separating flow and at  $c_a = 0$ , the value of  $c_m$  is indeterminate. Since the separation phenomena in the lower region for the most part proceed very slowly, unstable conditions develop in the practical field. There is agreement in the numerical values only for low-cambered profiles.

In addition to the simple relations between lift and drag and lift and wing moment, the gliding coefficient  $c_w/c_a$  and the climbing coefficient  $c_w/c_a^{1.5}$  are of interest to the constructor in his aerodynamic calculations. Corresponding fields were therefore plotted for these values, both for the pure profile values and for the assumptions of a particular example. Figures 8 and 9, respectively, represent a field for the gliding coefficient and for the climbing coefficient of the thickness group  $d/l = 0.1$  with an aspect ratio  $t/b = 1/5$  and a structural-drag coefficient  $c_{wr} = 0.03$ . It is apparent that the gliding coefficients gradually fall from the best value both with increasing and with decreasing  $c_a$  values, while the climbing coefficients, after passing the maximum, fall very rapidly with increasing  $c_a$  values.

From such curves the constructor can obtain a very good idea of the effect of varying the camber and also of varying the thickness in corresponding fields. If he selects, for example, in the fields 3, 8, 9, a particular profile and then changes his choice by taking a larger or smaller  $f/l$  ratio, he can im-

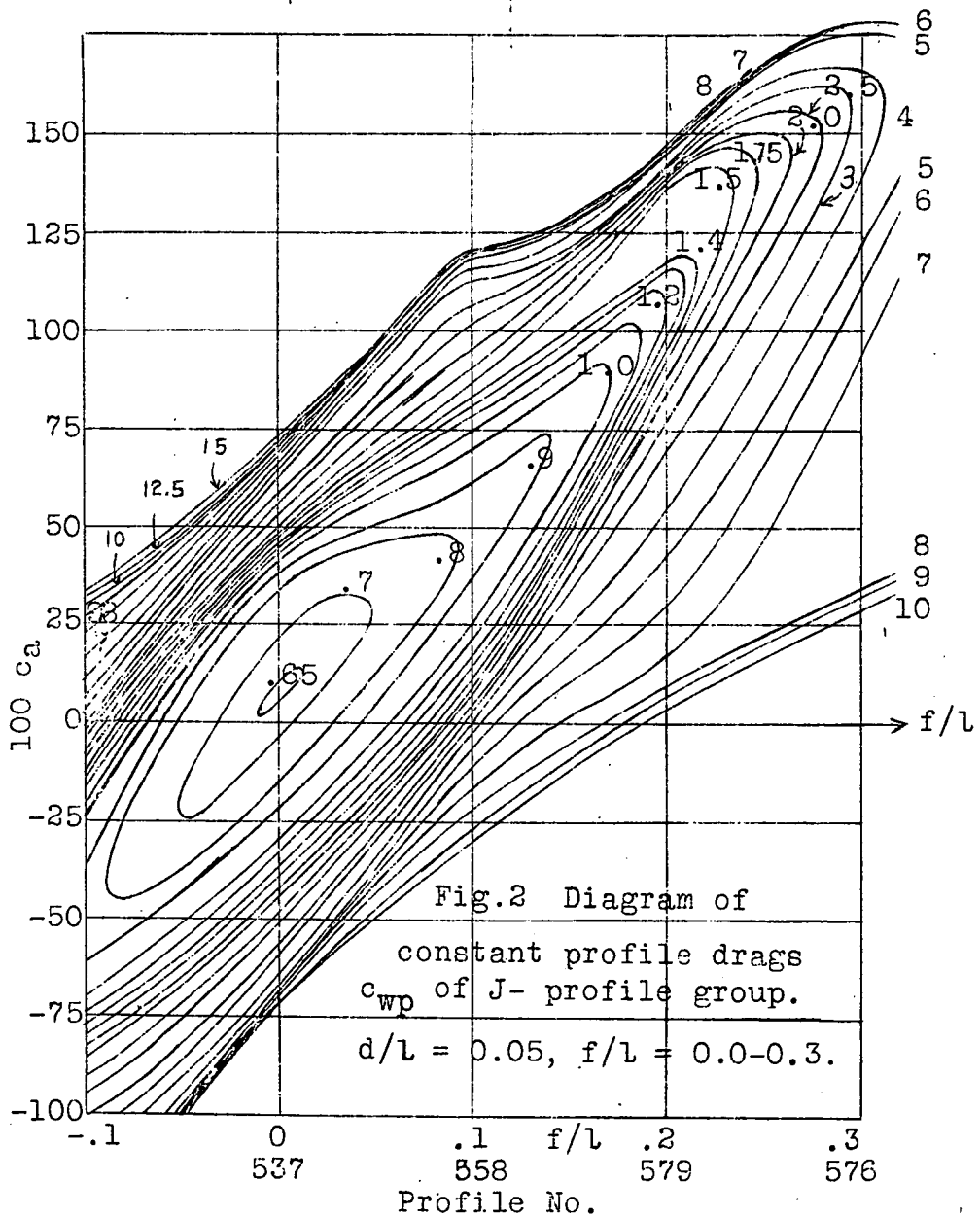
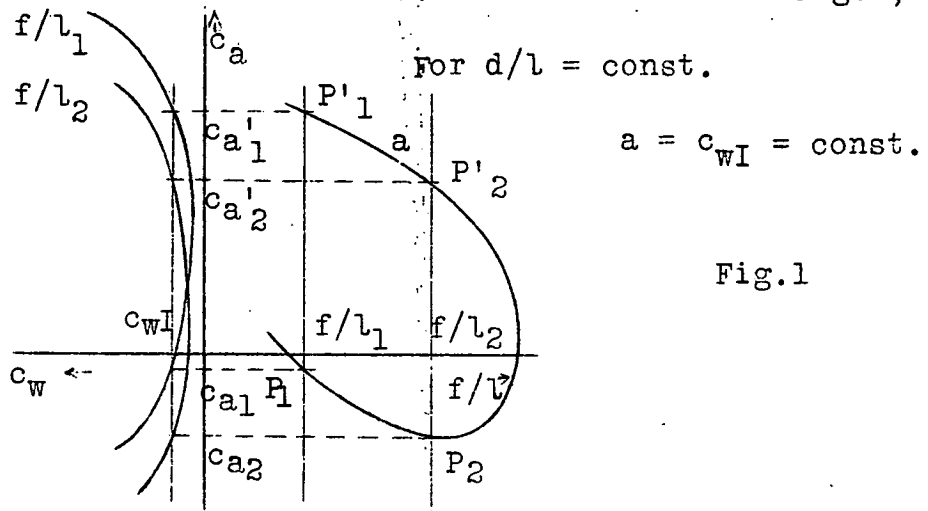
mediately see the effect of such changes. The practical range varies from  $f/l = 0.07$  to  $0.017$ . The  $c_{wi} + p_{min}$  varies thereby from  $0.0115$  to  $0.032$  and the  $c_{a \max}$  increases from  $1.25$  to  $1.5$ , while the gliding coefficient remains constant. The climbing coefficient gradually increases and then decreases. For airplanes built for speed, greater variations can be made toward the axis of symmetry, but the climbing coefficient becomes noticeably poorer. Conversely the climbing coefficient is fairly good up to  $f/l = 0.27$  and  $c_a$  increases to  $1.7$ , while the coefficient of glide rapidly grows poorer and  $c_{w \min}$  decreases to  $0.076$ . It should be remembered that profile thickness has very little effect. In any case it has much less effect than the camber. These curves afford valuable data for researches on variable camber wings. Further discussion of these problems falls outside the scope of this article, since their thorough discussion would necessitate a very exact investigation of the whole problem, both in its aerodynamic and its constructive aspects.

In commenting on these results, it may be said that the fields are sufficiently consistent. This means that the results are good, especially in consideration of the very great distorted scale. Difficulties first arise in the region of incipient separation, due to the very unstable condition of flow. The only profile which gave bad results throughout was No. 541, as already mentioned. The discrepancies between its old and new measurements are indicated by dotted lines (Fig. 3). The curves repre-

sent the purely experimental results and leave the rectification to the reader.

In the graphic method, it is probable that the interpolations will also give sufficiently accurate results.

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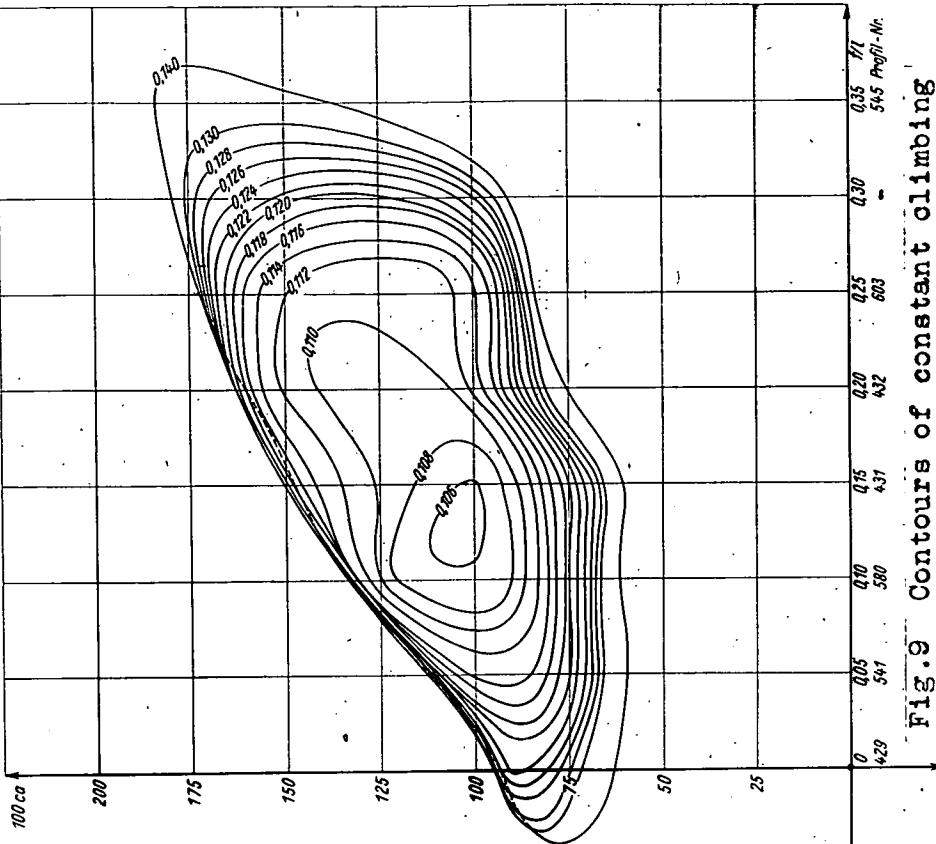


Fig.9 Contours of constant climbing coefficients  $c_w + c_{w1} + c_{wI}$  at

$$\lambda = \frac{1}{5} \text{ and } c_{wI} = 0.03 \quad c_a$$

of the J-profile group.  $d/l=0.1$ ,  $f/l=0.0-0.35$ .

(not rectified)

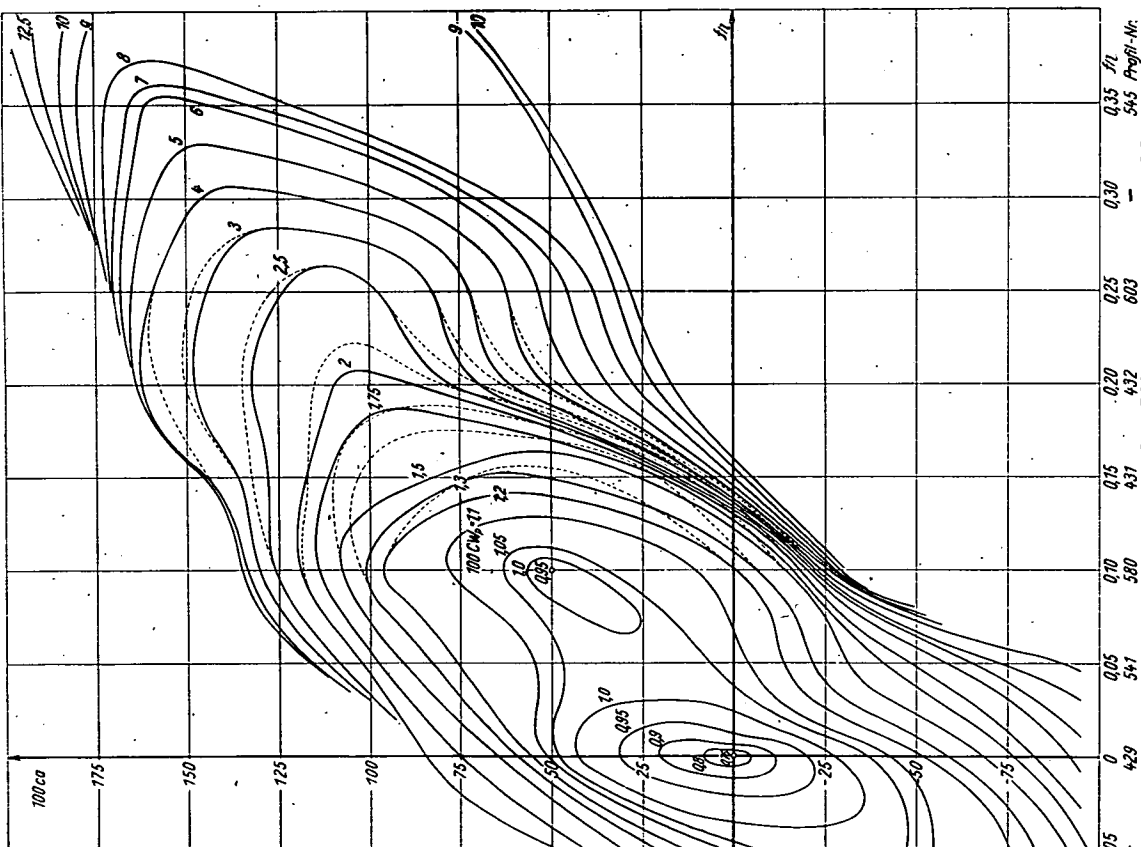
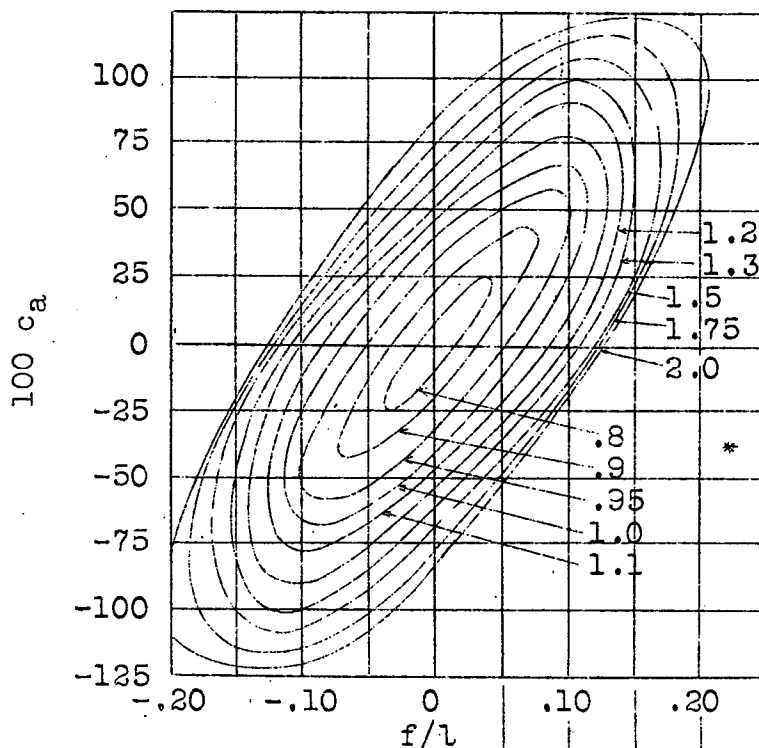


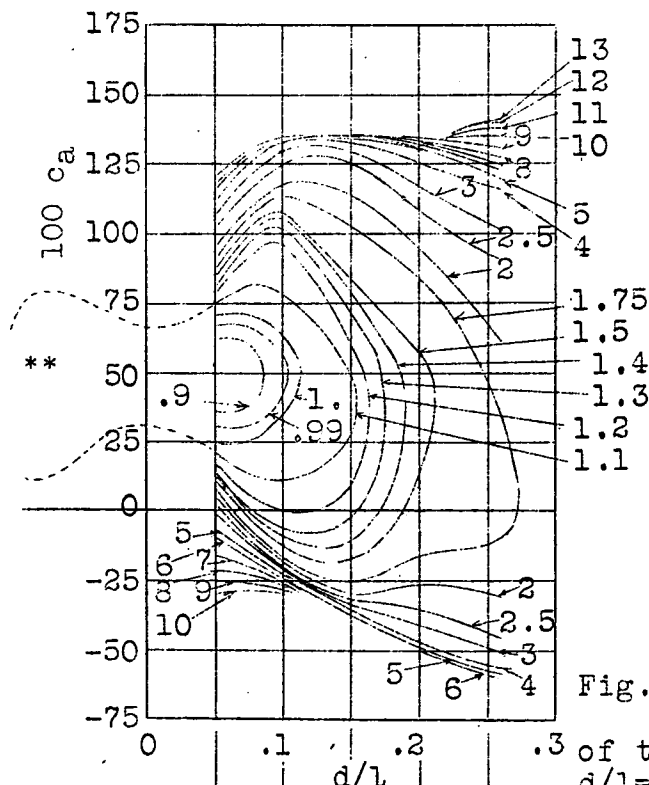
Fig.3 Diagram of constant profile drags of J-profile group. The dotted lines give results of new measurements.



\* 429 rectified symmetrically with reference to the origin. 541 disregarded, because very irregular.  $dc_{wp}/dc_a$  constantly increasing.

Profile No. 429 541 580 431 432

Fig.4 Corrected chart of constant profile drags of the J-profile group.  $d/l=0.1, f/l=0.0-0.35$ .



\*\* Example of an assumed course in the negative region. The negative thickness ( $-d/l$ ) corresponds to the positive thickness ( $+d/l$ ) according to the formula  $\frac{d_1}{l} = \frac{1}{l/d-2}$  (for  $\cos \gamma = 1$ ).

Fig.5 Chart of constant profile drags of the J-profile group.  $d/l=0.1, f/l=0.0-0.35$

Profile No. 558 580 433 434 435

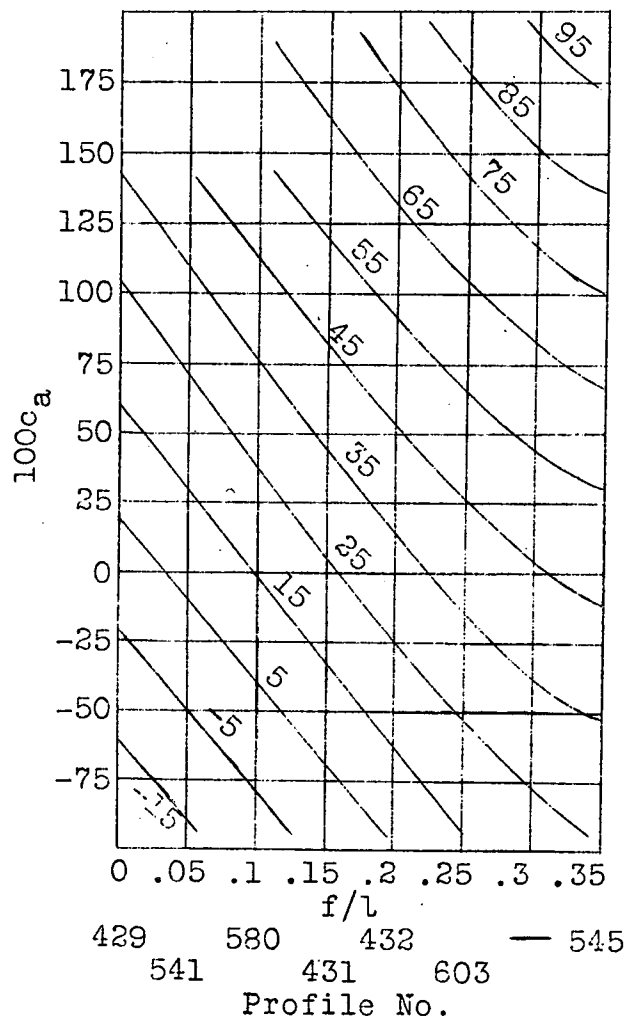


Fig.6 Diagram of constant theoretical  $c_m$  of J profile group.  $d/l=0.1$ ,  $f/l=0.0-0.35$



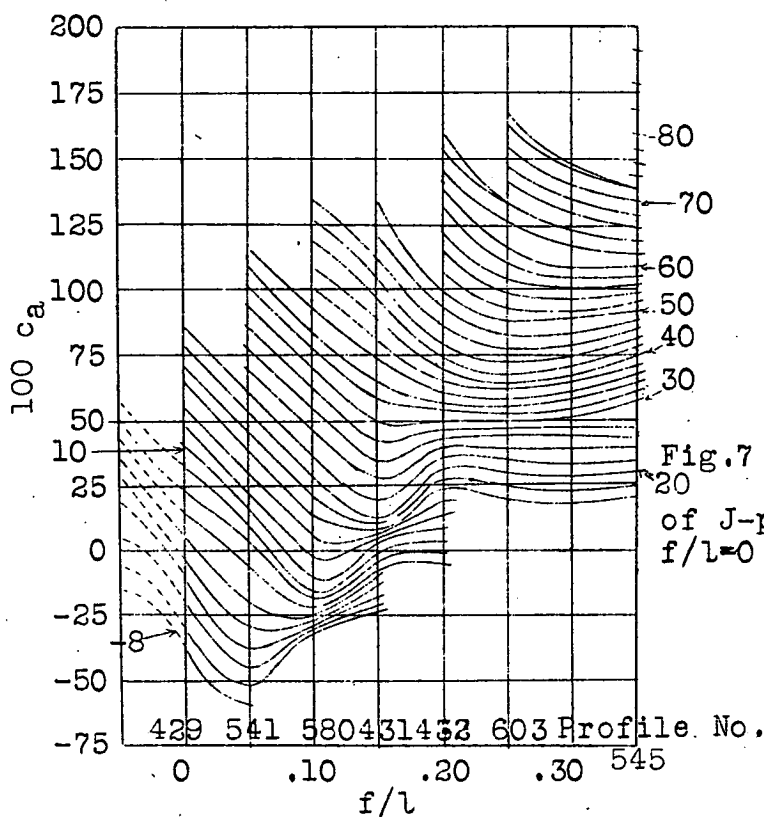


Fig. 7 Diagram of constant experimental  $c_m$  values of J-profile group.  $d/l=0.1$ ,  $f/l=0.0-0.35$ .

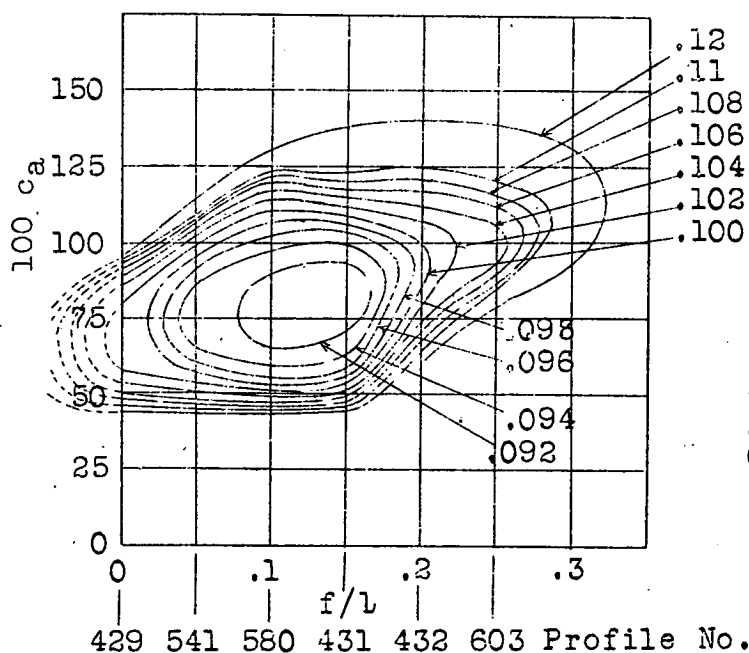


Fig. 8 Chart of constant gliding coefficients at  $\lambda=1/5$ ,  $c_{wr}=0.03$  for  $\epsilon=\epsilon_{\min}-\epsilon=0.12$  of J-profile group.  $d/l=0.1$ ,  $f/l=0.0-0.35$ .